

ECE 6564

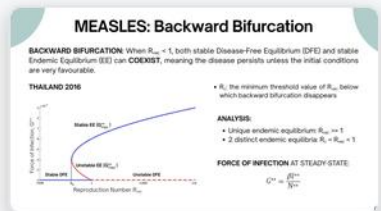
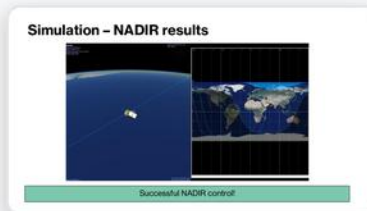
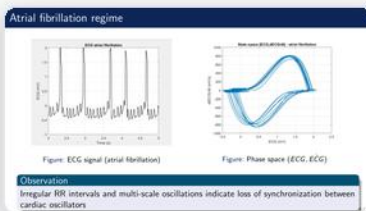
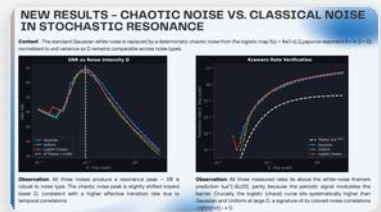
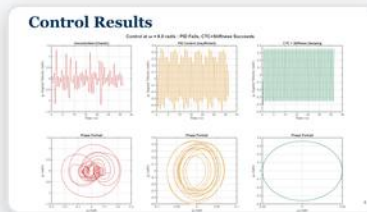
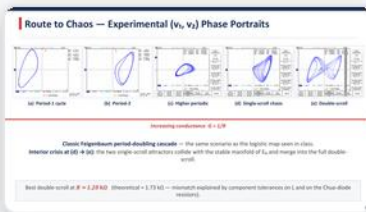
Nonlinear Dynamics and Chaos

Student Project Showcase

From abstract dynamics to physical, biological, and engineering systems

Spring 2025 - Spring 2026

Georgia Tech Europe / GT-CNRS IRL 2958

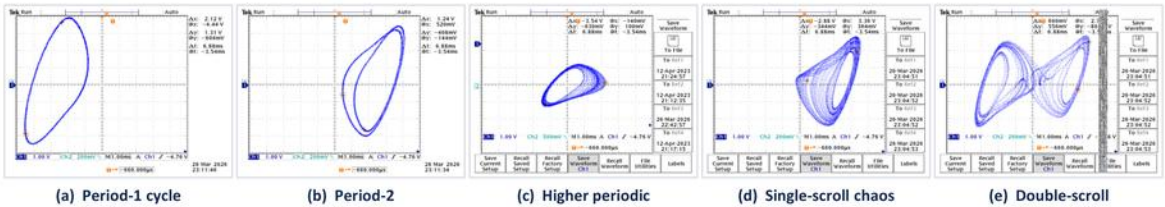


Chaotic circuits

Bifurcations, double-scroll attractors, and synchronization

- Students implemented Chua circuits and observed a route to chaos in phase portraits.
- The work connects theory, LTspice simulation, breadboard experiments, and chaos-masking communication.

Route to Chaos — Experimental (v_1, v_2) Phase Portraits



Increasing conductance $G = 1/R$

Classic Feigenbaum period-doubling cascade — the same scenario as the logistic map seen in class.

Interior crisis at (d) → (e): the two single-scroll attractors collide with the stable manifold of E_0 and merge into the full double-scroll.

Best double-scroll at $R \approx 1.28 \text{ k}\Omega$ (theoretical $\approx 1.73 \text{ k}\Omega$ — mismatch explained by component tolerances on L and on the Chua-diode resistors).

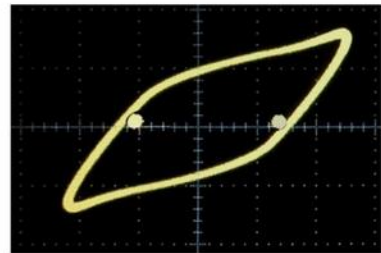
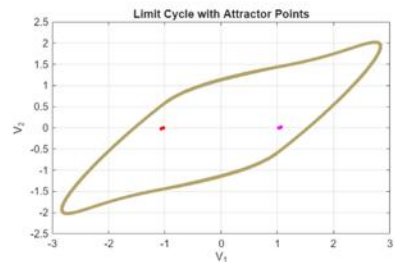
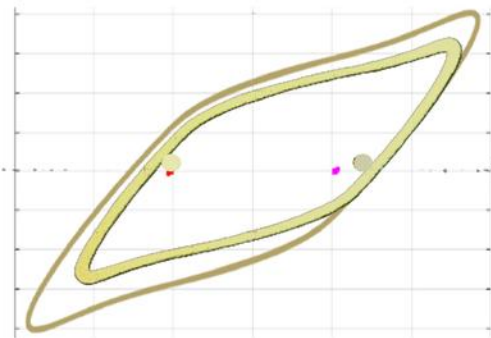
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Chaos in hardware

Modified Chua circuit and multistability

- A fourth-order Chua circuit was simulated and implemented physically.
- The project highlights coexisting attractors, component tolerances, and the gap between ideal models and breadboard behavior.

Result Comparison



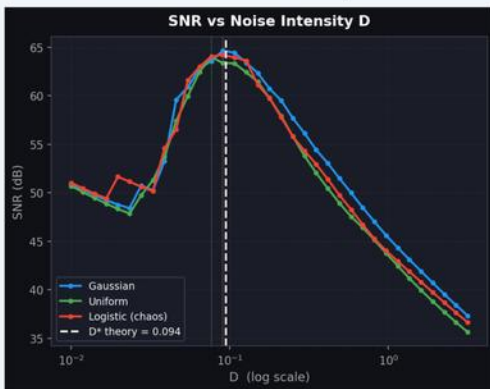
Stochastic resonance

When noise helps recover a weak signal

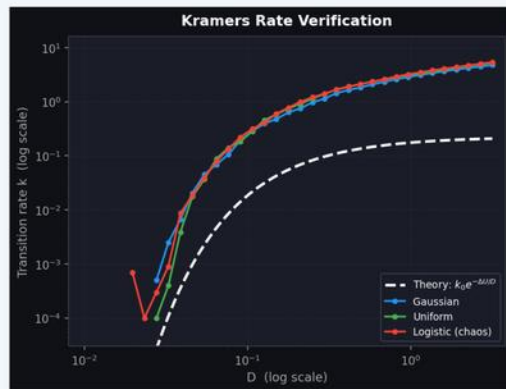
- Students compared Gaussian, uniform, and chaotic noise in a bistable nonlinear system.
- The project also included an STM32 / Schmitt-trigger experiment for weak-signal recovery.

NEW RESULTS – CHAOTIC NOISE VS. CLASSICAL NOISE IN STOCHASTIC RESONANCE

Context : The standard Gaussian white noise is replaced by a deterministic chaotic noise from the logistic map $f(x) = 4x(1-x)$ (Lyapunov exponent $\lambda = \ln 2 > 0$), normalized to unit variance so D remains comparable across noise types.



Observation: All three noises produce a resonance peak – SR is robust to noise type. The chaotic noise peak is slightly shifted toward lower D , consistent with a higher effective transition rate due to temporal correlations.



Observation: All three measured rates lie above the white-noise Kramers prediction $k_0 e^{-\Delta U/D}$, partly because the periodic signal modulates the barrier. Crucially, the logistic (chaos) curve sits systematically higher than Gaussian and Uniform at large D , a signature of its colored-noise correlations $\langle \eta(t)\eta(t+\tau) \rangle \neq 0$.

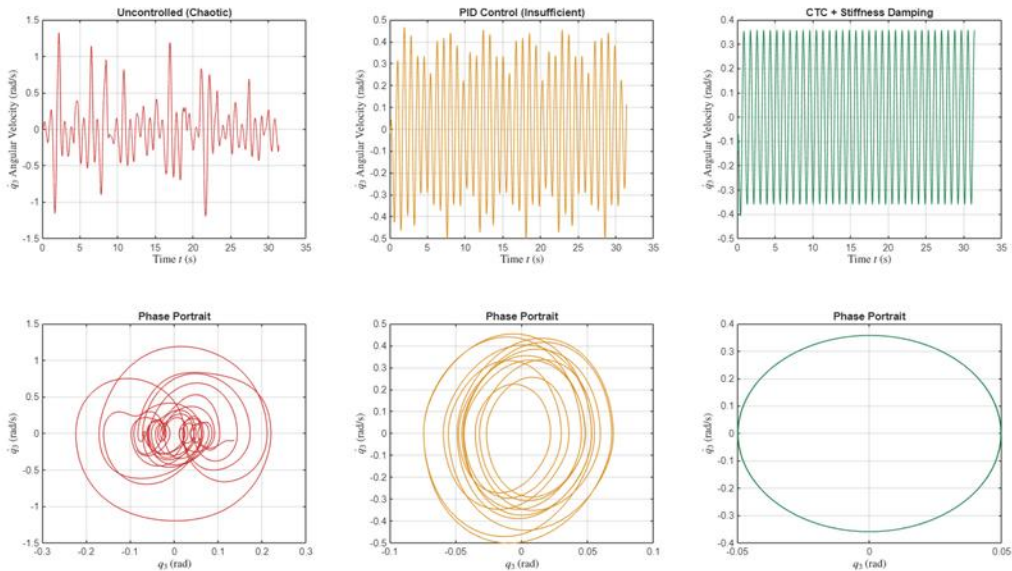
Robotics and control

Flexible joints, nonlinear elasticity, and chaos control

- A flexible 3-DOF robotic arm was used to explore how nonlinear mechanics can produce complex behavior.
- The project compared reactive PID control with model-based computed-torque control and stiffness damping.

Control Results

Control at $\omega = 6.0$ rad/s : PID Fails, CTC+Stiffness Succeeds



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Fluids and biology

The same mathematics in very different systems

- Chaotic stretching and folding can improve mixing in microfluidic devices.
- Coupled oscillator models can describe transitions between normal and irregular cardiac rhythms.

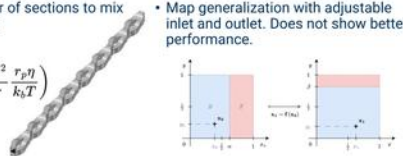
Mixing quality

- Required number of sections to mix two given fluids:

$$k \approx \frac{1}{2} \log_2 \left(6\pi \frac{\phi^2}{\tau} \frac{r_p \eta}{k_b T} \right)$$

- For honey $k=13$ (~2m)
- For ketchup and mustard $k=30$ (~6m)


• Map generalization with adjustable inlet and outlet. Does not show better performance.



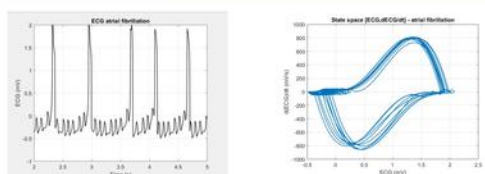
The diagram shows a mixing device with a central inlet and outlet, and a phase space plot with axes x and y . The plot shows a blue region and a red region, with a point $x_0 = \text{Final}$ marked. The Jacobian matrix is given as $J_f(x) = \begin{pmatrix} \lambda_x(x) & 0 \\ 0 & \lambda_y(x) \end{pmatrix}$.

$\forall x, \lambda_x(x) > 1$ i.e. $A_x(x) > 0$
 $\forall x, \lambda_y(x) < 1$ i.e. $A_y(x) < 0$

$D_L = 2$



Atrial fibrillation regime



The left plot shows an ECG signal with irregular RR intervals. The right plot shows the phase space (ECG, ECG) with a complex, multi-scale oscillation pattern.

Figure: ECG signal (atrial fibrillation)

Figure: Phase space (ECG, ECG)

Observation

Irregular RR intervals and multi-scale oscillations indicate loss of synchronization between cardiac oscillators

Chaotic mixing

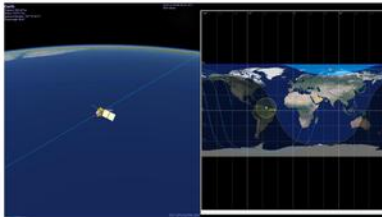
Cardiac rhythm dynamics

Space systems

Attitude control and orbital dynamics

- Spring 2025 projects connected nonlinear dynamics to mission-oriented space problems.
- Examples included satellite NADIR pointing and the circular restricted three-body problem.

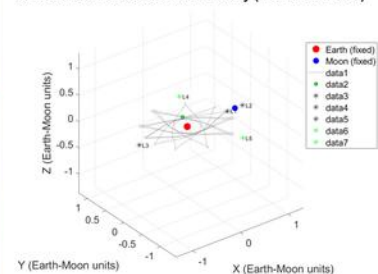
Simulation – NADIR results



Successful NADIR control!

Star Orbit Simulation

Animated Simulation of the 3rd Body (from saved data)



Satellite attitude control

Three-body problem

Epidemiological dynamics

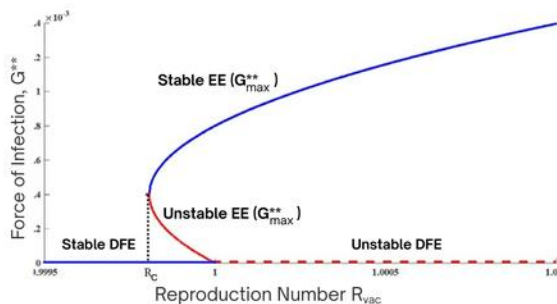
Thresholds, equilibria, and bifurcations in disease models

- Students used compartmental models to study tuberculosis, measles, and influenza A.
- The projects linked reproduction numbers, disease-free and endemic equilibria, and forward/backward bifurcations to control strategies.

MEASLES: Backward Bifurcation

BACKWARD BIFURCATION: When $R_{vac} < 1$, both stable Disease-Free Equilibrium (DFE) and stable Endemic Equilibrium (EE) can **COEXIST**, meaning the disease persists unless the initial conditions are very favourable.

THAILAND 2016



- R_c : the minimum threshold value of R_{vac} below which backward bifurcation disappears

ANALYSIS:

- Unique endemic equilibrium: $R_{vac} \geq 1$
- 2 distinct endemic equilibria: $R_c < R_{vac} < 1$

FORCE OF INFECTION AT STEADY-STATE:

$$G^{**} = \frac{\beta I^{**}}{N^{**}}$$

One mathematical language

Many systems. Shared concepts.

Bifurcations

Strange attractors

Synchronization

Lyapunov exponents

Poincaré maps

Multistability

Nonlinear control

Sensitivity to initial conditions

- Electronic circuits
- Robotics
- Microfluidics
- Cardiac rhythms
- Epidemiology
- Satellite control
- Orbital dynamics
- Communication